DISCRETE AND FINITE ELEMENT ANALYSES OF THE CONVENTIONAL TRIAXIAL TEST FOR GRANULAR MATERIALS

K. A. ABDEL-RAHMAN[1], M. ACHMUS[2]

ABSTRACT

From a theoretical point of view, the particle or Discrete Element Method (DEM) is more suitable for the modelling of the behaviour of granular materials like sand or gravel than approaches based on the continuum theory (FEM). But, due to the enormous numerical effort necessary for the DEM, the continuum theory is normally applied to model soil mechanical problems.

First, the results of Discrete Element Method (DEM) calculations of triaxial tests will be presented. The PFC^{3D} program was used to carry out these calculations. In this program the normally angular soil grains are modelled by spherical particles. Additionally, an up-scaling of the real particle diameters has to be carried out to enhance the calculation efficiency. By means of a comparison with experimental results of triaxial tests, a calibration for the input parameters of the model is done.

Second, the same triaxial tests using the same granular material will be modelled using the Finite Element Method (FEM). The constitutive model used is based on the theory of hypoplasticity. This special material law for granular soils simulates the non-linear stress-strain behaviour of the soil and also takes implicitly the stress-dependence of the angle of internal friction into account.

Comparisons between the results of Finite Element and Discrete Element simulations of triaxial tests for granular materials will be presented. Comparing the results of both FEM and DEM, hints and recommendations are given concerning the limits of both methods.

Keywords: Discrete Element Method, Finite Element Method, Hypoplasticity, Triaxial tests

[1] Dr.-Ing.; Institute of Soil Mechanics, Foundation Engineering and Waterpower Engineering, University of Hannover, Appelstr. 9A, D-30167 Hannover, Germany, Phone +49 511-762 2273; khalid@igbe.uni-hannover.de

[2] Prof. Dr.-Ing.; Head of the Institute of Soil Mechanics, Foundation Engineering and Waterpower Engineering, University of Hannover, Appelstr. 9A, D-30167 Hannover, Germany, Phone +49 511-762-4155; achmus@igbe.uni-hannover.de

INTRODUCTION

The object of this study is to compare between two methods used in solving geotechnical problems, first is the Discrete Element Method (DEM) which is mainly used for simulation of the behaviour of granular materials such as sand and gravel, second is the Finite Element Method which is widely used for analysis of many engineering problems of all disciplines. The comparison between the two methods and their suitability shall be assured throughout the results of laboratory tests.

DISCRETE ELEMENT METHOD

Discrete Element Methods are being increasingly used to simulate the mechanical behaviour of granular materials (Cundall & Strack 1979, Corkum 1986, van Baars 1996, Ni et al. 2000, Thornton 2000, McDowell & Harireche 2002). In the DEM the interaction between particles is regarded as a dynamic process achieving a static equilibrium when the internal forces are balanced. The dynamic behaviour is represented numerically by a time stepping algorithm using explicit time difference scheme.

This procedure of DEM takes advantage of the idea that the duration of the time step is selected and defined in a certain way, that during a single time step, disturbances in the state of equilibrium can spread only from the regarded particle to its direct neighbours. Each calculation cycle includes two stages: the application of simple interaction law at particle/particle or particle/wall contacts involving contact forces and relative displacements; and the application of Newton's Second Law of motion to determine the particle motion resulting from any unbalanced forces.

Each contact force has a normal and a tangential component calculated from the numerical overlapping of the particles using normal and tangential stiffness coefficients. A Coulomb type friction coefficient between particles limits the tangential contact forces. A similar behaviour is adopted for the particle/wall contact.

The DEM program used in this study is Particle Flow Code PFC^{3D} (Itasca 1995). The model used in PFC^{3D} can be regarded as a sub-class of the Distinct Element Method since it allows finite displacements and rotations of discrete bodies including detachment. It also recognizes new contacts automatically as the calculation progresses.

The program used has the following characteristics:
1 The particles are considered as homogeneous rigid balls.
2 The interaction between them is described as a soft contact, which occurs over an infinite small area.
3 The particles are allowed to overlap slightly at the contact points.
4 The slip condition between particles is governed by Mohr-Coulomb friction.

For the purpose of this study two more constraints are also applied:
1 The magnitude of the overlap is linearly related to the contact forces.
2 No tensile forces between particles are allowed.

Modelling of triaxial tests

For calibration, the numerical modelling has been performed simulating Karlsruhe medium sand to compare the numerical results with experimental ones. Karlsruhe sand consists mainly of subround quartz grains. The grain size distribution of this material is given in Fig. 1. The index properties of the sand are given in Table 1. The behaviour of Karlsruhe sand in triaxial tests was investigated by Kolymbas & Wu (1990). Results with dense samples ($e_0 = 0.53$) for different confining pressures (σ_3) are shown in Fig. 2.
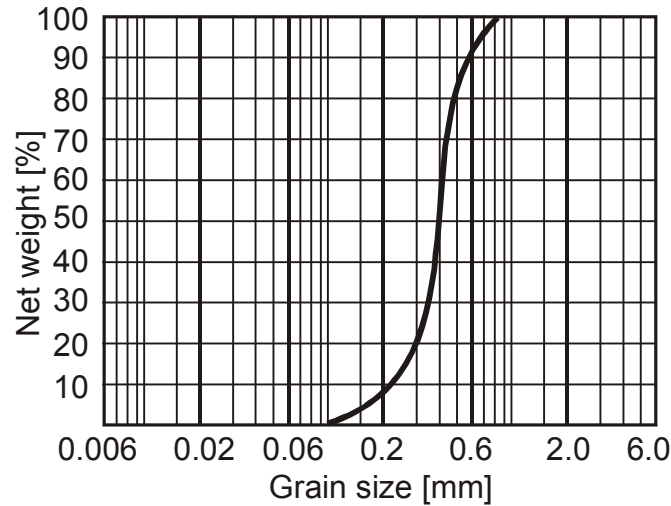

FIG. (1) Grain size distribution curve of Karlsruhe medium sand (Wu & Kolymbas 1991)

Table 1. Index properties of Karlsruhe medium sand (Wu & Kolymbas 1991)

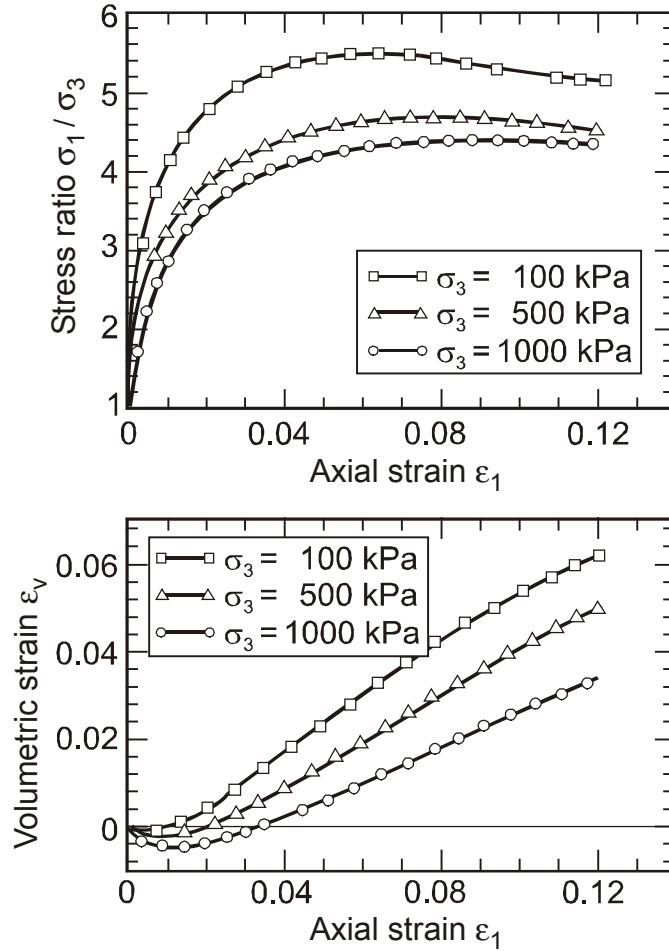|	
Unit weight of the grains, kN/m^3	26.5
D_{10}, mm	0.240
D_{60}, mm	0.443
Uniformity coefficient, C_u	1.85
Min. void ratio, e_{min}	0.53
Max. void ratio, e_{max}	0.84

FIG (2) Experimental triaxial test results for dense Karlsruhe medium sand (Kolymbas & Wu 1990)

For the numerical model, from practical point of view it is necessary to up-scale the grain diameter of the granular soil. Such a scaling process means the horizontal translation of the grain size distribution of the material. This leads to a smaller number of grains to enhance the computer efficiency to model the problem.

In this paper an up-scaling factor of 30 was chosen for the numerical model. The effect of up-scaling on the results of particle method calculations of granular materials was investigated by the authors (Achmus & Abdel-Rahman 2002).

The dimensions of the model were chosen with respect to experimental experience, which implies that the dimensions of the test sample should be greater than ten times the biggest grain diameter. In order to obey this rule, the dimensions of the model were chosen to be $25 \times 25 \times 25$ cm.

The sample of synthetic material in PFC^{3D} is represented as an assembly of spherical particles. The triaxial test was modeled by confining a cubic sample within six walls. The top and the bottom walls simulate loading platens and the lateral ones simulate the confining pressure experienced by the sample sides (Fig. 3). The sample is loaded in a strain-controlled fashion by specifying the velocities of the top and the bottom walls. Ideal test conditions were simulated by setting the friction coefficient between the sample and the walls to zero, thus avoiding any friction between the sample and the loading platens. During all the stages of the

test, the velocities of the lateral walls are controlled automatically by a numerical servo-mechanism (Itasca 1995) that maintains a constant confining stress within the sample.
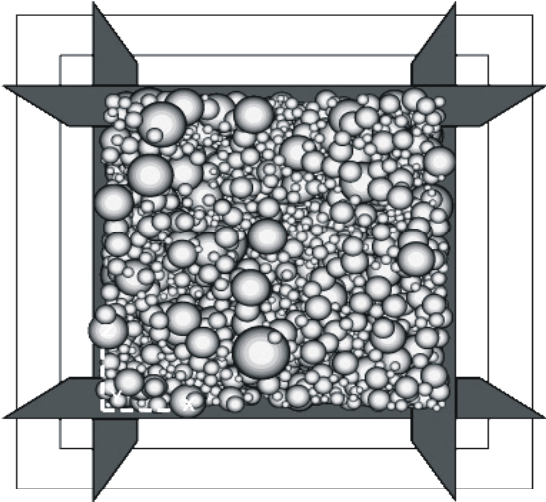

FIG. (3) Numerical model for triaxial test (top view, initial state)

During the calibration process the model parameters (normal stiffness k_n, tangential stiffness k_s and friction coefficient μ) were determined by comparing the experimental results with the numerical ones. The normal contact stiffness was chosen to $k_n = 2 \times 10^7$ N/m in order to match the results and to ensure that the overlaps between the particles are very small compared to the grain diameters.

The stress-strain behavior of the sample for small strains (i.e. in the quasi-elastic region) is mainly influenced by the ratio of the shear contact stiffness to the normal one. For the up-scaling factor usc = 30 a ratio of 1.0 was found to match best with the experimental results.

The peak stress ratio is dependent on the friction coefficient chosen. The higher the μ chosen, the higher the calculated peak stress. Best results were achieved with $\mu = 10$, a further increase did not yield a better agreement with the experimental results.

The numerical results obtained with the parameters reported in Table 3 for a confining pressure of $\sigma_3 = 100$ kN/m^2 are shown in Fig. 4.

Table (3) Microscopic parameters of the particle model

Normal stiffness k_n (N/m)	2×10^7
Tangential stiffness k_s (N/m)	2×10^7
Friction coefficient μ	10

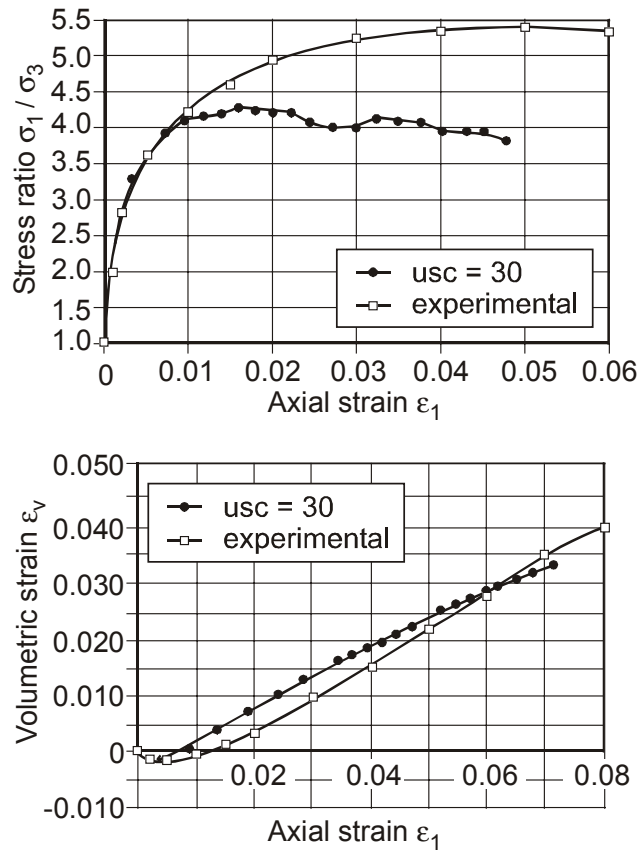FIG. (4) Stress ratio and volumetric strain plotted against axial strain for dense
Karlsruhe medium sand ($\sigma_3 = 100$ kN/m^2)

The results indicate that the non-linear stress-strain-behavior of sand including dilatancy is covered by the numerical model. However, even with the chosen high value of $\mu = 10$ the peak stress ratio found in the experimental test is not reached. Obviously, the effect of the particle shape (angularity of the real grains) can not be matched perfectly by increasing the friction coefficient for spherical particles.

Figure 5 gives numerical results for different confining pressures $\sigma_3 = 100$ kN/m^2 and $\sigma_3 = 500$ kN/m^2 obtained used the PFC^{3d} model. A qualitatively good agreement with the experimental results is found. Both the decrease of the peak stress level and the suppression of dilatancy with increasing confining pressure are obtained with the calculations.
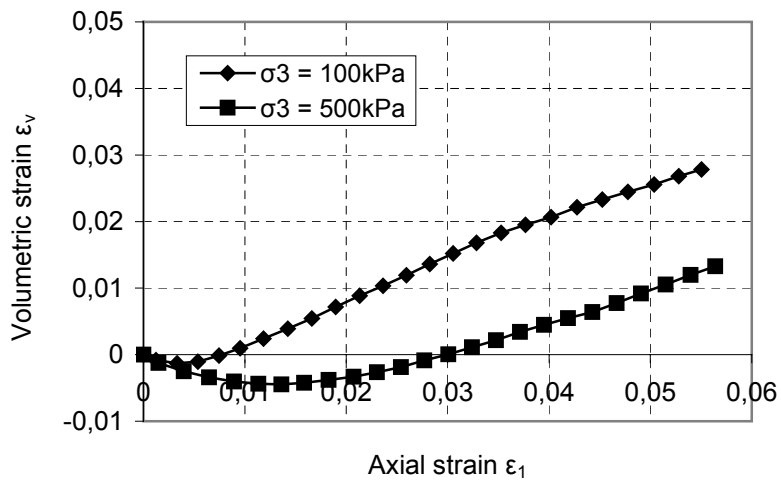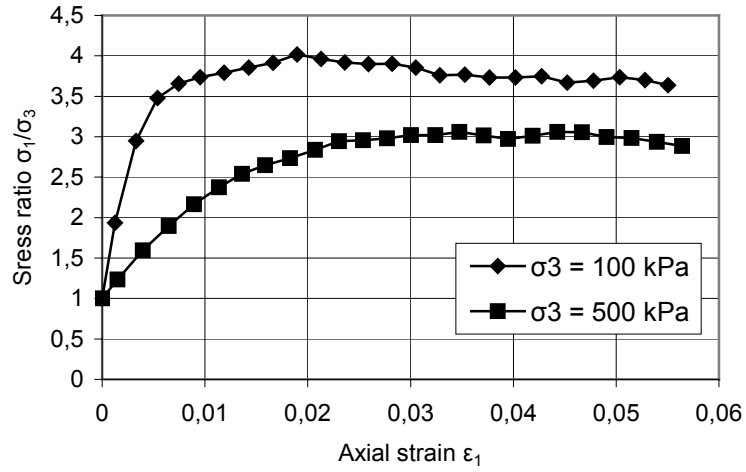
FIG. (5) Numerical triaxial test results for different confining pressures ($\sigma_3 = 100$ and 500 kN/m^2)

FINITE ELEMENT MODELING

Constitutive Equation

Hypoplasticity was introduced by Kolymbas (1977) in the general form

$$\mathbf{T_J} = \mathbf{h} \, (\mathbf{T}, \mathbf{D}, e) \tag{l}$$

with:

$\mathbf{T_J}$:   Jaumanns' stress rate
$\mathbf{h}$:   Tensor function
$\mathbf{T}$:   Cauchy stress tensor

D: symmetric part of the velocity gradient
e: porosity

The formulation for sands is rate independent, which means that the function **h** is positive homogeneous of first order in **D**. The constitutive equation allows for the application for large strain problems.

Abdel-Rahman (1999) used the same type of equation as von Wolffersdorf (1997) which was developed by Bauer (1996), Wu (1992), Gudehus (1996) in the form

$$\mathbf{T_J} = f_b \; f_e \; [\mathbf{L} \; (\mathbf{T}^*, \mathbf{D}) + f_d \; \mathbf{N} \; (\mathbf{T}^*) \; \|\mathbf{D}\|] \tag{2}$$

with tensor functions **L** and **N**. $\|\mathbf{D}\|$ is the euclidian norm. The functions f_e and f_d describe the influence of density and f_b the influence of mean pressure. **T*** is the normalized Cauchy stress tensor using the trace of **T**. The constitutive equation has 8 constants, which partly are connected in a simple way to standard tests in soil mechanics. The eight constants used in the finite element analysis are calibrated for the so called Karlsruhe sand see therefore Table 4. For a detailed discussion about the mathematical background and physical meaning of the input parameters for the constitutive model refer to Herle (1997).

Table (4) The input parameters for Karlsruhe sand

φ_c=30.0°	h_s=5800MN/m^2	e_{d0}=0.53	e_{c0}=0.84
n=0.25	e_{i0}=1.00	α=0.13	β=1.05

Numerical Model

In the finite element analysis the program-system ABAQUS (version 6.3) was used. The dimensions of the model were chosen in such a way to be similar to Discrete Element Model. For the finite element computations isoparametric continuum elements with 4 nodes were used. The axial symmetrical condition is assumed. The initial state was the hydrostatic pressure applied to the sample ($\sigma_2 = \sigma_3$).
For the boundary conditions, the nodes along the symmetrical axis are allowed to move only in the vertical direction while the nodes along the bottom boundaries are restrained against vertical as well as horizontal movements.
Then the vertical stress (σ_1) was applied to the sample under constant confining pressure. The numerical results obtained for different confining pressures (σ_3 = 100, 500 and 1000 kN/m^2) are shown in the Figure 6.
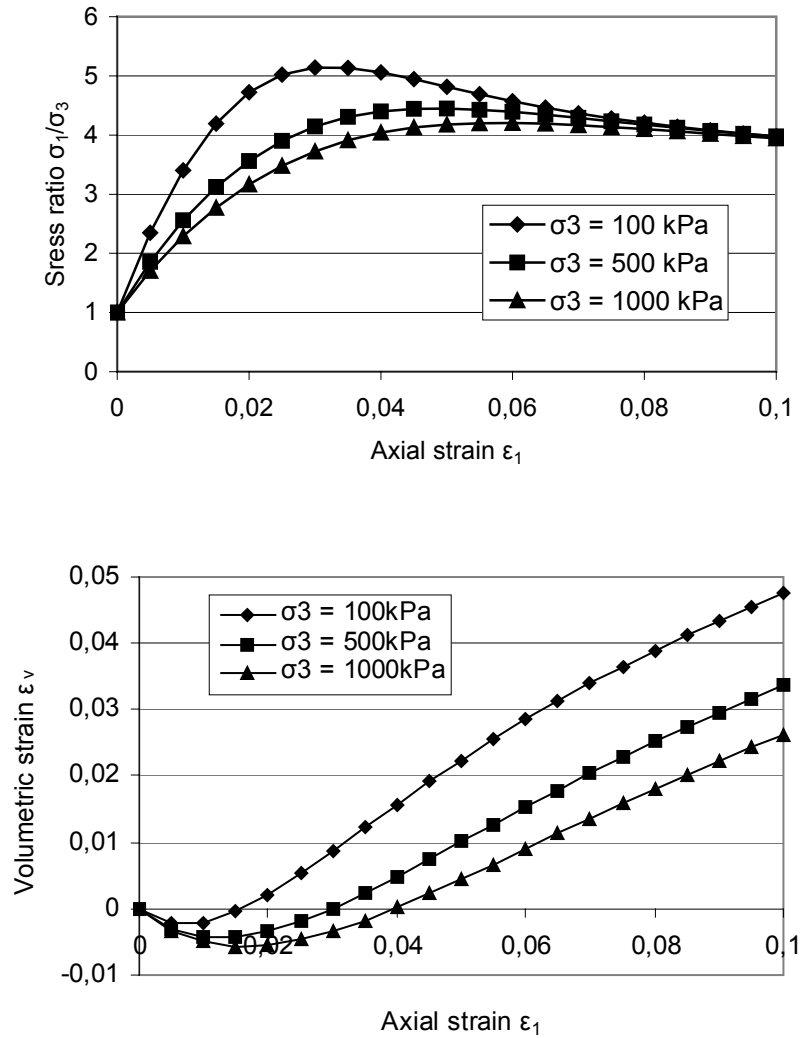
FIG. (6) Finite element results of triaxial tests for dense Karlsruhe medium sand ($e_0 = 0.53$)

The results indicate a remarkable good agreement of the numerical and the experimental tests (Fig. 2).The calculated peak stress levels agree with the experimental results. The dependence on the confining pressure and thus the stress level-dependance of the angle of internal friction is obtained. This is a special feature of the hypoplastic material law used, for which no explicit angle of internal friction has to be defined.

Also, the suppression of dilatancy with increasing confining pressure observed in the experiment is obtained with the numerical model. The amount of volumetric strain agrees very good with experimental values.

COMPARISON BETWEEN DISCRETE AND FINITE ELEMENT RESULTS AND CONCLUSIONS

In the following figure (Fig. 7) the results obtained with the discrete and the finite element calculations are compared as an example for a confining stress of $\sigma_3 = 500$ kN/m^2.
Both the methods generally reflect the nonlinear behaviour of sand including the dilatant behavior of the dense sand modelled.
The hypoplastic material law is a kind of a "state of the art"-approach for the macroscopic modelling of granular materials. Very good quantitative agreement of the numerical results with the experimental ones (not shown in the figure) is obtained.
The discrete element method used herein is found to considerably underestimate the peak stress level, whereas the simulation of the dilatant behavior is in good agreement with the experiments as well as with the finite element results.
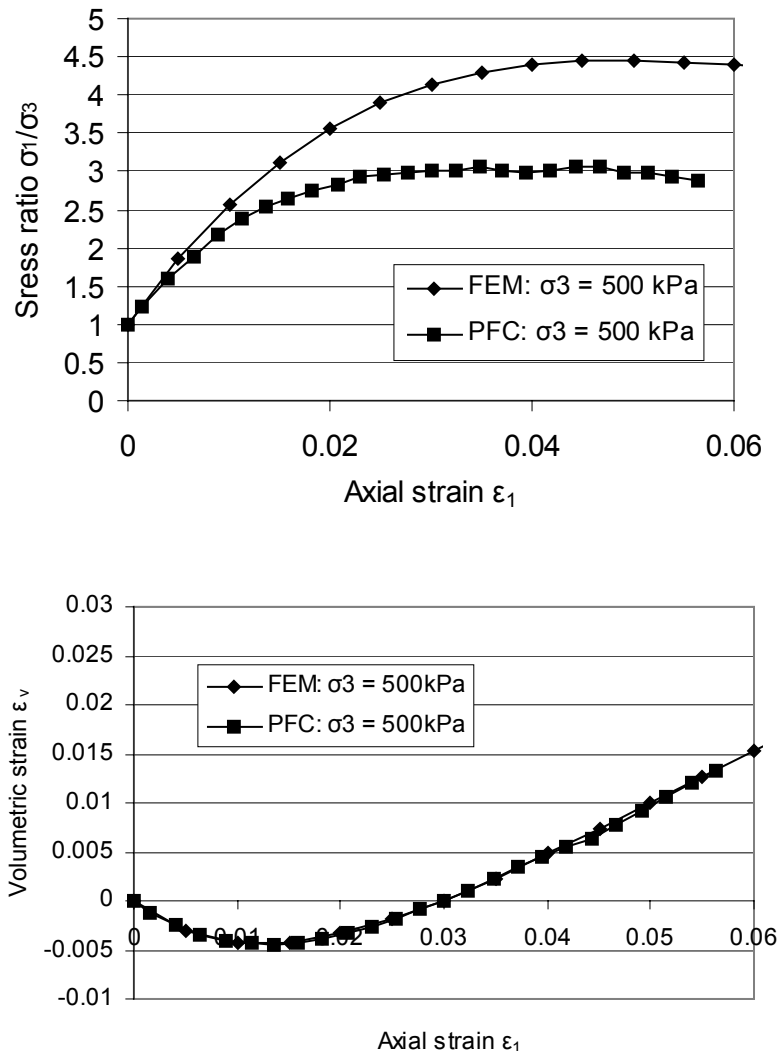

FIG. (7) Comparsion between the results of discrete element and finite element Method for triaxial test

The reason for the underestimation of peak stresses lies probably in the simulation of the real, angular sand particles by perfect spheres in the PFC model. Obviously, the real shear strength cannot be matched perfectly by simply increasing the coefficient of friction in the particle model. Thus, it has to be stated that for better agreement with experimental results the particle model should be improved by considering non-sherical particles.

The main conclusions of the presented analyses are as follows:
1 Concerning a qualitative aspect, the highly non-linear behaviour of sand in both element tests is covered by both the methods.
2 The discrete element model underestimates peak stresses. This is probably due to the idealization of the angular grains by spherical particles in the model used.
3 The finite element model using the hypoplastic material law reflects the real behaviour of the sand in the triaxial test.
4 Further analysis is necessary for a better understanding of the limits of both methods. Problems with different stress paths and especially with non-monotonic and cyclic loading have to be considered to evaluate the possibilities and limitations of the methods.

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