

# Numerical modelling of the combined axial and lateral loading of vertical piles

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**ABSTRACT:** In offshore technology large diameter steel pipes are often used for pile foundations. Especially when used as foundation elements for offshore wind energy converters, such piles experience not only vertical, but also substantial horizontal loading. In this paper the interaction between axial and lateral forces acting on vertical steel pipe piles in sand is investigated by means of numerical modelling. A three-dimensional finite element model was developed with a non-linear elasto-plastic material law for the sand. It is found that for axial tension loads the decrease of axial pile stiffness due to horizontal loads has to be taken into account. For axial compression loads as well as for horizontal loading the interaction effects are of minor importance and can be neglected in the design.

## 1 INTRODUCTION

Tripod or jacket foundations can be applied for offshore wind energy converters planned in the German part of the North Sea at large water depths from 30 up to 50 m. For these structures, driven steel pipe piles are used as foundation elements. The piles are driven through a pile sleeve fixed at the corner points of the foundation structure. After pile driving, the connection is carried out by means of grouting.

The loads applied to these quasi-vertical piles differ greatly from typical onshore and also offshore structures, with more substantial lateral loading. These loads result from the combined actions of vertical dead load and horizontal loads due to wind and wave action.

Current design practice involves separate analysis of the axial and lateral responses of piles and does not consider the effect of interaction between the different load directions. This paper deals with the interaction effects for piles embedded in sand, since in the German North Sea sand soils are to be expected in most cases.

Several results of investigations concerning the behaviour of piles in sand subjected to combined axial and lateral loading (inclined loading) were reported in the literature, e. g. by Yoshimi (1964), Das et al. (1976), Chari & Meyerhof (1983), Ismael (1989) and Sastry & Meyerhof (1990). In some investigations combined horizontal load and vertical compression load are concerned, in others oblique tensile loads are considered. From these investigations, it seems that the pile response to

horizontal loading is only slightly affected by a vertical load, whereas horizontal loads significantly affect the vertical pile response. This was also determined by Ismael (1989), who investigated the behaviour of bored piles subjected to axial and oblique pulls.

In this paper, the results of a numerical study on the behaviour of vertical piles in sand under inclined compression and tension loads are presented. For this a three-dimensional finite element model was used.

## 2 FINITE ELEMENT MODELLING

For the investigation of the behaviour of vertical piles under combined loading conditions a three-dimensional (3-D) numerical model was developed. The finite element programme ABAQUS (Abaqus 2005) was applied. An advanced computer system with parallel processor technology was used to minimize the computation time.

Steel pipe piles with a length of 20.0 m and a wall thickness of 2 cm were considered. Two different outer diameters  $D = 2.0$  and  $D = 3.0$  m were chosen in order to study the effect of pile geometry.

The most important issue in geotechnical numerical modelling is the simulation of the soil's stress-strain-behaviour. An elasto-plastic material law with Mohr-Coulomb failure criterion was used to describe the behaviour of medium dense sand. The soil stiffness is represented here by a stiffness modulus for oedometric compression  $E_s$  and a

Poisson's ratio  $\nu$ . To account for the non-linear soil behaviour, a stress dependency of the stiffness modulus was implemented as follows:

$$E_s = \kappa \sigma_{at} \left( \frac{\sigma_m}{\sigma_{at}} \right)^\lambda \quad (1)$$

Here  $\sigma_{at} = 100 \text{ kN/m}^2$  is a reference (atmospheric) stress and  $\sigma_m$  is the current mean principal stress in the considered soil element. The parameter  $\kappa$  determines the soil stiffness at the reference stress state, and the parameter  $\lambda$  rules the stress dependency of the soil stiffness.

The material parameters used in the calculations are given in Table 1.

Table 1. Material parameters used for medium dense sand.

Unit buoyant weight $\gamma'$	11.0 kN/m <sup>3</sup>
Oedometric stiffness parameter $\kappa$	400
Oedometric stiffness parameter $\lambda$	0.60
Poisson's ratio $\nu$	0.25
Internal friction angle $\varphi'$	35°
Dilation angle $\psi$	5°
Cohesion $c'$	0.1 kN/m <sup>2</sup>

The material behaviour of the piles was assumed to be linear elastic with the parameters  $E = 2.1 \cdot 10^5 \text{ MN/m}^2$  (Young's modulus) and  $\nu = 0.2$  (Poisson's ratio) for steel. The considered pile geometries are given in Table 2.

Due to the symmetric loading condition only a half-cylinder representing the sub-soil and the pile could be considered. The discretized model area had a diameter of twelve times the pile diameter. The bottom boundary of the model was extended by six times the pile diameter below the base of the pile. With these model dimensions the calculated behaviour of the pile is not influenced by the boundaries (Fig. 1).

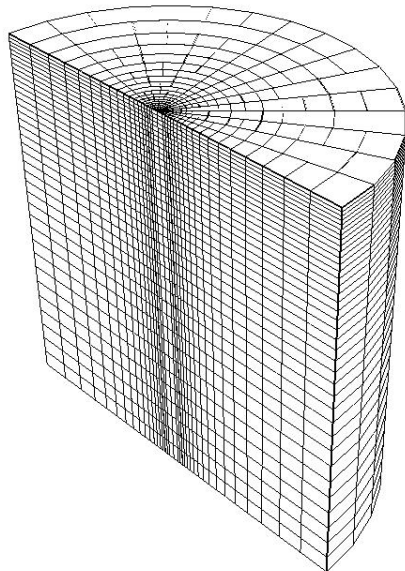


Figure 1. Finite element model.

Table 2. Pile properties.

Case	Diameter	Wall thickness	Length
	$D$	m	$L$
1	2.0	0.02	20.0
2	3.0	0.02	20.0

8-node continuum elements were used for the soil as well as for the pile. The frictional behaviour in the boundary surface between pile and soil was modelled by contact elements, where the wall friction angle was set to  $\delta = 0.67 \varphi'$ .

The finite element calculation is executed stepwise. At first, for the generation of the initial stress state the whole model area is discretized using soil elements only. Subsequently, the pile is generated by replacing the soil elements located at the pile position by steel elements and activating the contact conditions between pile and soil.

Finally, the vertical load and the horizontal load are applied simultaneously and increased gradually until the required maximum loads are reached.

The piles were subjected to loads with various inclinations ( $\alpha = 0.0, \pm 30.0^\circ, \pm 60.0^\circ, \pm 90.0^\circ$ ) measured from the horizontal direction, whereby the positive sign of  $\alpha$  stands for compression loads and the negative sign for tension loads (Fig. 2).

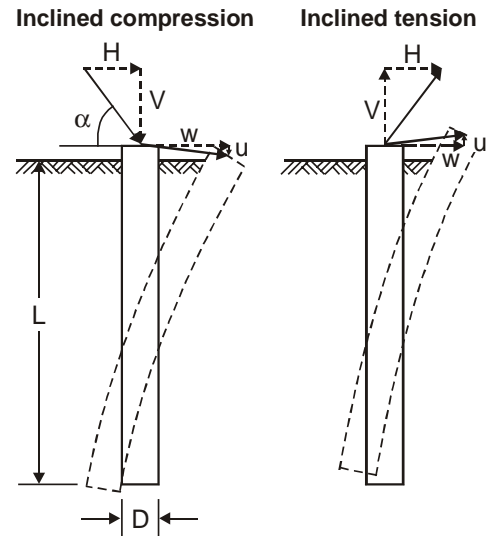


Figure 2. System and denominations.

### 3 NUMERICAL MODELLING RESULTS

#### 3.1 Generalities

In order to investigate the influence of combined axial and lateral loading, the load inclination  $\alpha$  was varied in seven steps between  $-90^\circ$  (pure axial compression) over  $0^\circ$  (pure horizontal loading) to  $90^\circ$  (pure axial tension). For the problem concerned, the pile head displacements in the horizontal and vertical direction are of major interest. A comparison of the  $H$ - $w$ - and  $V$ - $u$ - curves (ref. to Fig. 2) clearly shows the effect of the vertical load on the

horizontal load-deformation behaviour and vice versa. Hence, these curves are given in the following for different load inclinations. In this respect, a distinction is made between inclined compression load (i.e. axial compressive load and horizontal load) and inclined tension load.

### 3.2 Results for piles under inclined compression loads

The horizontal load deformation behaviour for the pile with a diameter of 2 m is shown in Fig. 3. Due to these results, the horizontal displacement is nearly independent of the load inclination and thus independent of a vertical load acting together with the horizontal load. Similar results were found experimentally by Sastry & Meyerhof (1990), who carried out model tests with inclined compression loads.

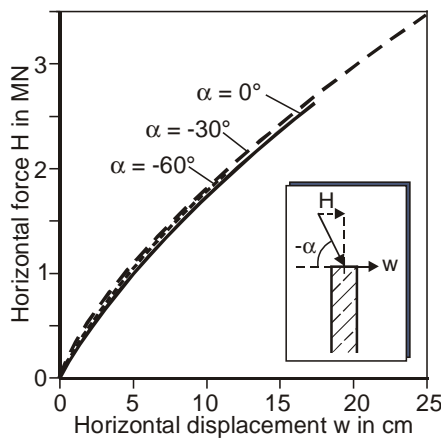


Figure 3. Horizontal displacement at the pile top dependent on horizontal load (inclined compression,  $D=2.0\text{m}$ ,  $L=20.0\text{m}$ ).

Instead, the vertical displacement (settlement) of the pile is affected by a horizontal load. In Fig. 4 the calculated dependence of settlement and vertical load for different load inclinations is given for the pile with a diameter of 2 m. The horizontal load has a favourable effect, since it leads to a stiffer behaviour in the vertical direction. For instance, for a vertical force of 2 MN the settlement is reduced by about 25% for  $\alpha = -60^\circ$  and even by about 50% for  $\alpha = -30^\circ$  compared to the pure compression case.

Figs. 5 and 6 show the load-displacement curves for a pile with a diameter of 3 m. There is a tendency for similar results as for the case  $D = 2\text{ m}$  to be obtained. For the larger diameter, a small stiffening effect of the horizontal behaviour is obtained with increasing axial load. However, this effect is of minor importance.

### 3.3 Results for piles under inclined tension loads

In Fig. 7 the horizontal load-displacement behaviour with variable axial tensile loads is presented for the

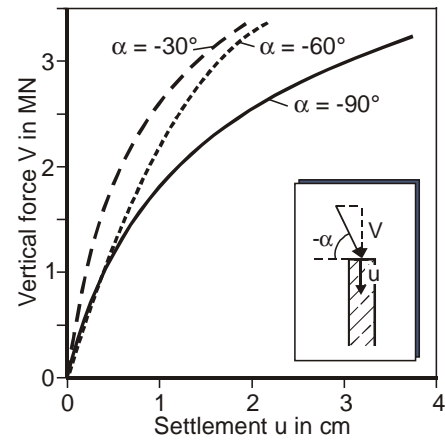


Figure 4. Settlement of the pile top dependent on vertical load (inclined compression,  $D=2.0\text{m}$ ,  $L=20.0\text{m}$ ).

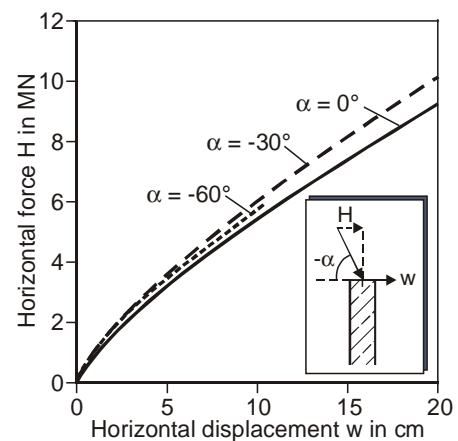


Figure 5. Horizontal displacement at the pile top dependent on horizontal load (inclined compression,  $D=3.0\text{m}$ ,  $L=20.0\text{m}$ ).

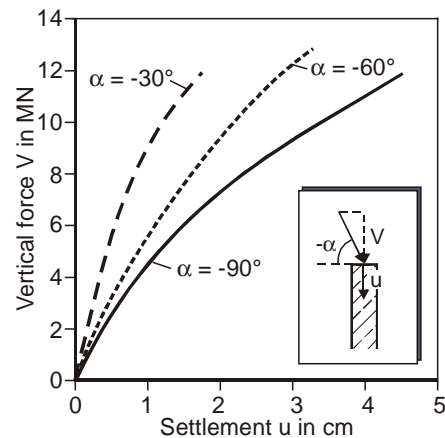


Figure 6. Settlement of the pile top dependent on vertical load (inclined compression,  $D=3.0\text{m}$ ,  $L=20.0\text{m}$ ).

pile with a diameter of 2 m. As for the inclined compression case, at first there is no significant influence of the vertical load on the  $H-w$ -curve. But, from a certain load level which is dependent on the load inclination, the curves for inclined loads deviate from the curve for pure horizontal loading. Larger horizontal displacements then apply, i.e. the horizontal pile stiffness is decreased.

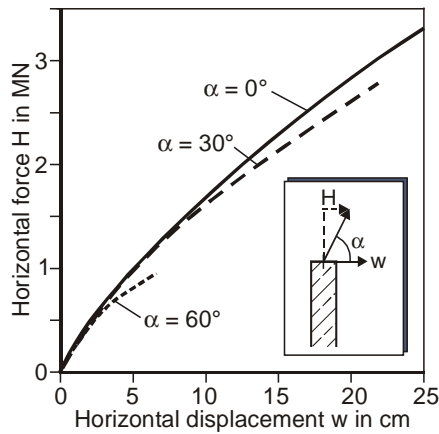


Figure 7. Horizontal displacement at the pile top dependent on horizontal load (inclined tension,  $D=2.0\text{m}$ ,  $L=20.0\text{m}$ ).

The respective vertical load-displacement curves for the case with  $D = 2\text{ m}$  are shown in Fig. 8. Here again, as for the inclined compression load, a significant influence of the horizontal load is found. The vertical pile stiffness is distinctly reduced when compared to the case with pure axial tension. But, on the other hand, a horizontal load increases the ultimate vertical pile capacity. Thus, the unfavourable effect of decreased stiffness is joined by the favourable effect of increased capacity.

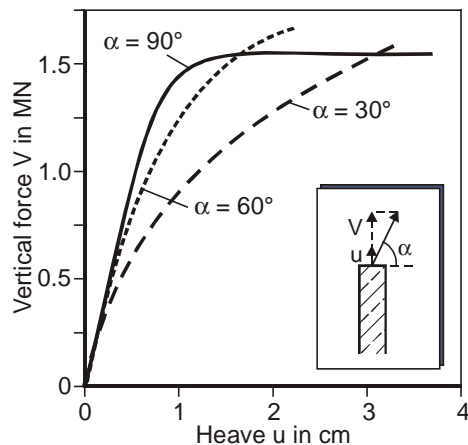


Figure 8. Pile heave dependent on vertical load (inclined tension,  $D=2.0\text{m}$ ,  $L=20.0\text{m}$ ).

Figs. 9 and 10 show the load displacement curves obtained for the pile with a diameter of 3 m under inclined tension loads. The results tend to be similar, which means that the general behaviour described above applies independent of the pile diameter.

The vertical load, above which a significant impact on the horizontal load-deformation behaviour begins, corresponds to the load above which a distinct increase of the pile heave is to be noticed. Thus, the reason for the deviation in the  $H-w$ -curves is obviously that the pile capacity for tension load is smaller than the horizontal pile capacity in the cases considered. If the vertical load approaches the ultimate load, this ultimate load becomes decisive for the combined ultimate load.

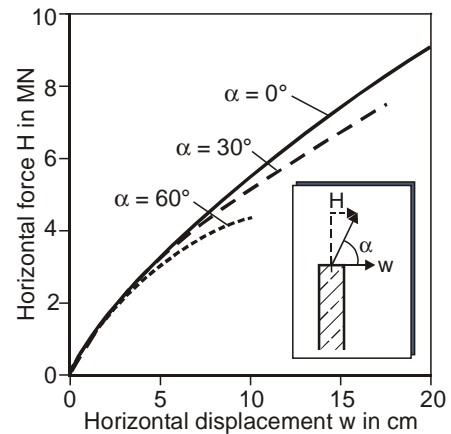


Figure 9. Horizontal displacement at the pile top dependent on horizontal load (inclined tension,  $D=3.0\text{m}$ ,  $L=20.0\text{m}$ ).

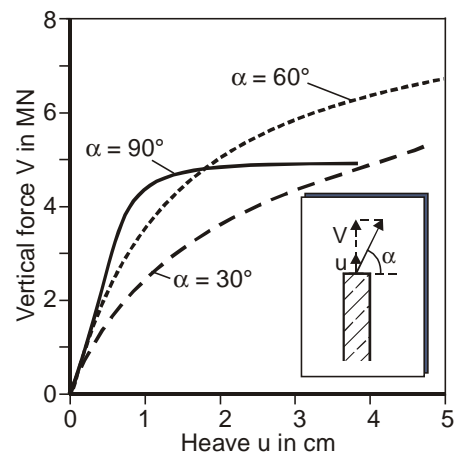


Figure 10. Pile heave dependent on vertical load (inclined tension,  $D=3.0\text{m}$ ,  $L=20.0\text{m}$ ).

## 4 ANALYSIS OF THE NUMERICAL RESULTS

### 4.1 Piles under inclined compression loads

The results from the numerical modelling show a positive effect of a horizontal load on the pile settlement behaviour. A substantial reason for this is that in the upper part of the pile large horizontal stresses act on the ‘passive’ side in front of the pile due to the horizontal load. On the passive side bedding reactions are induced, which are much larger than the horizontal stresses in the ‘at rest’ state. Compared to this, the decrease of horizontal stresses on the active side (behind the pile) is a minor effect. The same applies below the point of rotation of the pile, with the stress increase now on the other side of the pile.

This is elucidated by the presentation of the vertical shear stresses acting on the pile in the plane of symmetry, i.e. in the loading plane, given in Fig. 11. The upper part gives the results for  $\alpha = -30^\circ$  and a load level, at which the mobilizable skin friction (horizontal load multiplied by the coefficient of friction  $\tan \delta$ ) is not fully utilized. The shear stresses behind the pile (on the active side) are very low, but

high shear stresses act in front of the pile. These stresses are much higher than the maximum shear stresses for pure axial loading, which can be assumed to increase linear with depth  $z$  according to the equation  $\tau_{max} = \gamma z k_0 \tan \delta$ .

The lower part of Fig. 11 gives the shear stress distributions for  $\alpha = -60^\circ$  and a higher loading level. The mobilizable shear stresses on both sides of the pile are fully utilized. Hence, the resultant shaft resistance of the pile is significantly larger than for the case of pure compression load.

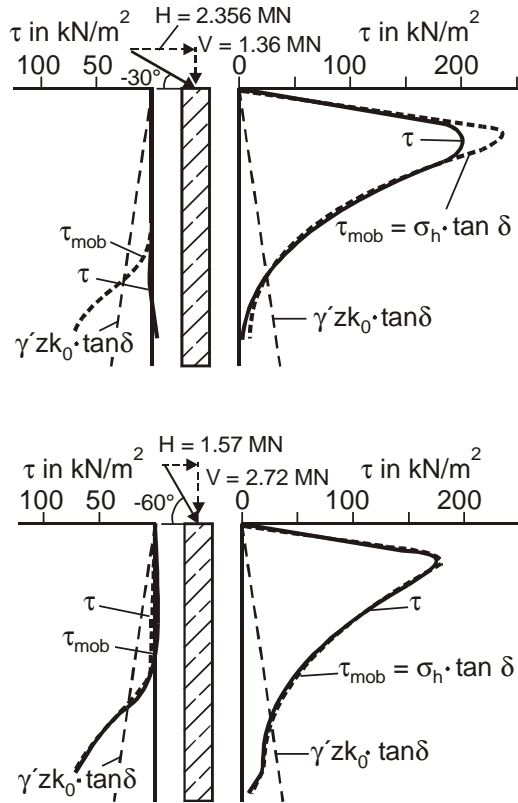


Figure 11. Shear stress distribution along the pile in the loading plane for two inclined compression loading cases ( $D = 2.0$  m,  $L = 20.0$  m).

Another reason for the increased vertical pile stiffness obtained is a prestress effect due to the horizontal load. In the region near the soil surface in front of the pile, where the maximum shear stresses occur, the soil is moved upwards by the horizontal load. To carry the vertical load, a pile needs a relative displacement to mobilize the skin friction. This displacement is partly induced by the horizontal load, i.e. fewer pile settlements are necessary to carry a certain load.

#### 4.2 Piles under inclined tension loads (oblique pull)

For vertical tension loads, the interaction between horizontal and vertical loads leads to unfavourable effects. The vertical pile stiffness and – beyond a certain load level – also the horizontal pile stiffness is reduced.

The stiffness decrease in the vertical direction results from negative skin friction induced by the horizontal load. This is elucidated in Fig. 12, where the shear stresses acting on the pile in the loading plane are shown for a load inclination of  $\alpha = 60^\circ$  and two different loading levels. The horizontal displacement of the pile corresponds to a vertical upwards directed displacement of the soil body in front of the pile. This displacement is larger than the pile heave due to tension load, so that negative skin friction, i.e. an additional tensile load on the pile, occurs. Accordingly, the pile heave is increased.

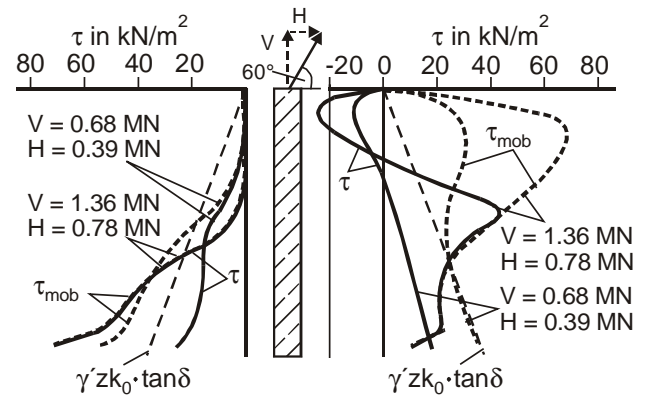


Figure 12. Shear stress distribution along the pile in the loading plane for inclined tension loading cases ( $D = 2.0$  m,  $L = 20.0$  m).

In principle, negative skin friction is only a problem of serviceability, i.e. deformations, but not a problem of ultimate capacity. If a large pile heave occurs due to high tensile load, the direction of the relative displacements between pile and soil is reversed, so that ‘positive’ shear stresses can be utilized. Thus, the ultimate load for the combined action of vertical and horizontal load is also larger for the inclined tension load case than for the case with pure tensile loading, since the mobilizable skin friction is increased by the horizontal force. However, the pile heaves belonging to the ultimate state are much larger than the heave belonging to the ultimate capacity for pure tension loading (see Figs. 8 and 10).

Ismael (1989) also stated that theoretically larger vertical pile capacities occur under inclined tension load. However, he mentioned that this was never observed in field tests. The reason for that could be that this capacity increase will only be observed for very large displacements, which may not have been reached in field tests.

In Fig. 13 the results obtained are presented in the form of an ‘oblique pull’ curve. Here the displacement in the direction of loading is given dependent on the resultant load. This kind of presentation is usually chosen when the behaviour of piles for the anchoring of moorings is considered.

The result is that the anchoring becomes the stiffer, the larger the load inclination, i.e. the vertical

portion of the load, is. However, the resultant capacity becomes the lowest for pure vertical loading. This was also found experimentally by Das et al. (1976).

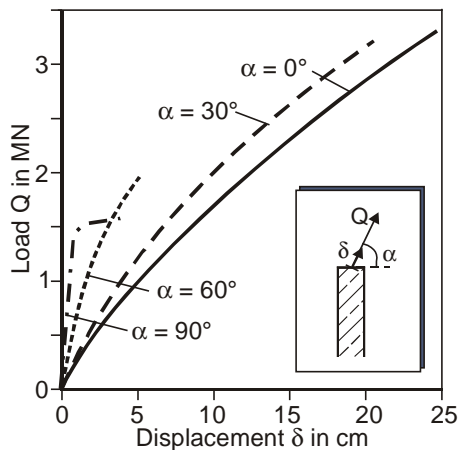


Figure 13. Load-displacement curves for oblique pull ( $D = 2.0$  m,  $L = 20.0$  m).

However, the piles considered here and also the piles investigated by Das et al. are relatively short with ratios of pile length to pile diameter of around 10. Due to this, the vertical capacity of the piles is lower than the horizontal capacity. If a long pile with a higher vertical than horizontal capacity is considered, the stiffness as well as the capacity of the mooring anchor increases with the load inclination  $\alpha$  (see, for example, Ismael 1989).

## 5 CONCLUSIONS

According to the numerical calculation results, piles in sand under combined axial compression and horizontal load behave more favourably than purely horizontally or purely axially loaded piles.

The horizontal pile stiffness is almost unaffected by a vertical compression load acting simultaneously. Thus, the design of pile bending with usual methods without consideration of interaction (e.g. p-y curve method) seems justified.

In the vertical direction with a compression load an increase of pile stiffness and capacity is induced by a horizontal load. The pile design without consideration of the interaction effects thus lies on the safe side.

Different conditions apply for combined axial tension and horizontal load. The vertical pile stiffness is significantly reduced by a horizontal force. This must be taken into account in the pile design. However, in principle this is a problem of negative skin friction, and hence not a problem of ultimate pile capacity.

For the design in the horizontal direction, interaction effects can also be neglected for axial tension loads. Since the actual tension load has sufficient safety related to the ultimate tension load

(determined without interaction effects), no significant influence on the horizontal stiffness occurs. This is ensured by the pile design in the axial direction.

To summarize the results, the interaction between horizontal and vertical load must only be considered in the determination of axial displacements due to tension loads.

However, the question of how cyclic loads affect the interaction behaviour remains open. The loads on foundations of offshore wind energy converters are of extremely cyclic nature. For piles of tripod and jacket structures, the horizontal as well as the vertical load are to a great extent cyclic, with the latter even varying between tension and compression. This leads to an accumulation of displacements and to a change of the load-bearing behaviour in both directions. For instance, in the horizontal direction a reduction of soil stiffness with cyclic loading is to be expected.

The effect of cyclic loading on the interaction behaviour should be investigated. A numerical or analytical analysis alone is not sufficient here, i.e. experimental tests are indispensable.

## 6 ACKNOWLEDGMENTS

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